

## The Political Business Cycle: A Comment

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Nordhaus' recent paper in this *Review* [1] is of particular interest as it offers an explicit analysis of the interaction between the economy and the polity, albeit for a partial section of the economy only. It analyses both the steady-state equilibrium and the disequilibrium paths within electoral periods. This comment is directed to the first part only, where the long-run choice in democratic systems over the course of many electoral periods is studied. Nordhaus reaches the "fundamental long-run result" that:

"The democratic outcome corresponds to the policy which was found . . . to be purely myopic. That is, it comes at the point where the implicit rate of time preference is infinite" [1, p. 179].

This implies that a democratic government chooses a point on the long-run Phillips-curve that has lower unemployment and higher inflation than optimal.

This comment is intended to show that Nordhaus' result is not an attribute of a democratic government as such, but rather depends on the *specific* assumptions made about the government's utility function. If another—*equally plausible*—utility function is used, quite different results follow. Using the same assumptions about voters' evaluations of unemployment and inflation and about the trade-off between the two, it turns out that unemployment is always higher, and inflation correspondingly lower, than Nordhaus suggests to be typical for a democracy.

The economic model is unchanged, but instead of the goal of *vote-maximization* [1, p. 174] it is assumed that a government wants to *maximize the length of time* which it can expect to remain uninterruptedly in power. It seems plausible that government politicians are not only interested in winning the next election but do in fact look beyond it, especially if they are confident of winning the forthcoming election. The younger party members will exert particular pressure in this direction as they are interested in ensuring that their party is still popular with the voters in the future, when they have a chance to take over governmental jobs.<sup>1</sup>

To formalize this idea, the aggregate voting function  $g_n(-u, -\pi)$  may be interpreted as the *government's probability of election* (or re-election) in period  $n$  ( $0 \leq g_n < 1$ ). A government may stay in power over periods  $t = 1, 2, \dots, \infty$ . The probability of being in power in period 1 and *not* being elected in period 2 is  $g_1(1-g_2)$ ; the probability of being in power both in period 1 *and* 2, but of being defeated in period 3 is  $g_1 \cdot g_2(1-g_3)$ , etc. In general, the probability of being in power uninterruptedly over  $n$  periods and of being defeated in period  $n+1$  is  $\prod_1^n g_t(1-g_{n+1})$ . The expected length of time in power is the sum of the various periods of being in government, weighted by the above probability

$$\sum_{t=1}^{\infty} t(1-g_{t+1}) \prod_{\tau=1}^t g_{\tau} \quad \dots(1)$$

This expression may also be derived by assuming a discrete utility function taking the value one if the party is in power, and zero otherwise. The expected utility of  $n$  uninterrupted periods in government is  $n(1-g_{n+1})\prod_1^n g_t$ . The sum of expected utilities over all possible government periods is equation (1). Using the assumption that a democratic government

is never absolutely sure of whether it will be re-elected (i.e.  $g < 1$ ), equation (1) can be simplified to

$$\sum_{t=1}^{\infty} \prod_{\tau=1}^t g_{\tau} \quad \dots(2)$$

Putting  $\prod_{\tau=1}^t g_{\tau} = Z_t$ , we have as the maximand

$$\sum_{t=1}^{\infty} Z_t, \text{ with } Z_1 = g_1, Z_t = g_t Z_{t-1}. \quad \dots(3)$$

To simplify the formal analysis, a continuous version of (3) is used:

$$\int_0^{\infty} Z(t) dt, \text{ with } Z(0) = g(0) \text{ and } \dot{Z}(t) = [g(t) - 1]Z(t), \quad \dots(4)$$

where  $\dot{Z}(t) \equiv dZ/dt$ .

Maximizing (4), subject to the economic model (see [1], equations (12) and (13))

$$\pi_t = f(u_t) + \lambda v_t, \quad \dots(5)$$

$$\dot{v}_t = \gamma(\pi_t - v_t) \quad \dots(6)$$

and restricting to the steady-state solution ( $\dot{v} = \dot{u} = 0$ ), determines a democratic government's *optimal choice* between unemployment and inflation:

$$\frac{f'(u)}{1-\lambda} = -\frac{g_1}{g_2} \left( \frac{(1-g) + \gamma(1-\lambda)}{(1-g + \gamma)(1-\lambda)} \right), \quad \dots(7)$$

with  $0 \leq \lambda < 1$ .  $g_i$  indicates the partial derivative of  $g(\cdot)$  with respect to the  $i$ th argument.

This solution may be directly compared with Nordhaus' general optimum [1, equation (14)]

$$\frac{f'(u)}{1-\lambda} = -\frac{g_1}{g_2} \left( \frac{\rho + \gamma(1-\lambda)}{(\rho + \gamma)(1-\lambda)} \right), \quad \dots(8)$$

with  $0 \leq \lambda < 1$ . For a democratic government, the implied rate of time preference  $\rho$  is infinite, and equation (8) reduces to the *purely myopic policy* [1, equation (16M)]

$$f'(u) = -g_1/g_2. \quad \dots(9)$$

Equations (7) and (8) differ only by the fact that the rate of time preference  $\rho$  is substituted in (7) by  $[1 - g(-u, -\pi)]$ . The implicit discount rate used by a government interested in staying in power as long as possible thus corresponds to the probability of *not* being re-elected: a government expecting to be defeated discounts the future heavily; a government confident of being re-elected discounts the future correspondingly little. It should be noted that the probability of defeat  $(1-g)$  *explicitly* depends upon  $u$ , while in Nordhaus' approach the value of the discount rate  $\rho$  is determined implicitly only. The two optima (7) and (9) for democratic governments do *in general* indicate *different* choices of unemployment and inflation. The assumption of quasi-concavity of  $g(\cdot)$  also used by Nordhaus, and the usual convexity assumption of  $f(\cdot)$  allow us to draw a clear conclusion concerning the relationship of the various "optimal" rates of unemployment. If the unemployment rate according to the Golden Rule (see [1], equation (16G))

$$f'(u)/(1-\lambda) = -g_1/g_2 \quad \dots(10)$$

is added, it may be shown (see appendix) that

$$u_G > u_D > u_N. \quad \dots(11)$$

The rate of unemployment chosen by a democratic government as here proposed ( $u_D$ ) is (for  $\lambda > 0$ ) always *larger* than Nordhaus suggests ( $u_N$ ) and *smaller* than the Golden Rule ( $u_G$ ). The opposite holds, of course, for the rate of inflation. The differences between the various optima rise with increasing  $\lambda$ .

In contrast to Nordhaus' "fundamental long run result" for democratic governments, it may no longer be maintained that

"democratic systems will choose a policy on the long-run trade-off that has lower unemployment and higher inflation than optimal" [1, p. 178].

Nordhaus' result follows from his *specific* choice of the government's objective function: It is after all not surprising that a government whose time horizon extends only up to the next election is purely myopic because in long-run equilibrium the time periods in between elections are insignificant. The only points of interest are the election dates which follow each other without interruption. *A purely myopic policy is not a general characteristic of governments in democracies.*

APPENDIX

Proof of (11):

Equations (7), (9) and (10) may be interpreted as special cases of

$$f'(u) - (1-\lambda)R(u)z = 0, \tag{**}$$

with

$$f'(u) < 0, \quad f''(u) \geq 0, \\ R(u) \equiv -g_1/g_2 < 0, \quad R'(u) < 0.$$

For  $z = 1$  follows (10), for  $z = [(1-g) + \gamma(1-\lambda)] / (1-g + \gamma)(1-\lambda)$  follows (7), and for  $z = 1/(1-\lambda)$  follows (9). As  $0 \leq g(\cdot) < 1, \gamma > 0,$

$$1 < [(1-g) + \gamma(1-\lambda)] / (1-g + \gamma)(1-\lambda) < 1/(1-\lambda). \tag{**}$$

Implicitly differentiating (\*) and using the assumptions mentioned in the text

$$\frac{du}{dz} = \frac{(1-\lambda)R(u)}{f''(u) - (1-\lambda)R'(u)z} < 0. \tag{***}$$

This proves (11).

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NOTE

1. As Nordhaus' specification, this objective function gives no weight to the expected utility derived from returning to power after a period out of office (see [1], p. 178, footnote 2). This would, however, lead to a very complex differential game.

REFERENCE

[1] Nordhaus, William D. "The Political Business Cycle", *Review of Economic Studies* (April 1975), 169-190.

