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AND GOVERNMENT BEHAVIOR

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## IDEOLOGY, PUBLIC APPROVAL, AND GOVERNMENT BEHAVIOR

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In most macroeconomic models, the government policy variables (or instruments) are treated as exogenous variables.<sup>1</sup> However, casual observation reveals that government economic policy does change in response to the performance of the economy as well as to external stimuli such as currency devaluation. Hence the policy variables are in fact endogenous variables in a much broader context. In this paper an attempt is made to integrate the behavior of the government as an endogenous part of an economy.<sup>2</sup>

The basic behavioral hypothesis is that the government (or party)<sup>3</sup> attempts to maximize its utility, which is a function of certain goals or welfare indicators of the economy, subject to the condition that it is reelected at every election. The government in power can adjust the levels of these indicators, subject to given exogenous conditions, through the policy variables. The performance of the economy, in turn, influences the number of votes the incumbent party may receive. The situation may be represented schematically in Figure 1.

This approach departs from the more traditional quantitative studies of economic policy-making, as no social welfare function per se is postulated.<sup>4</sup> Instead, the government is assumed to possess a complete preference ordering over the set of

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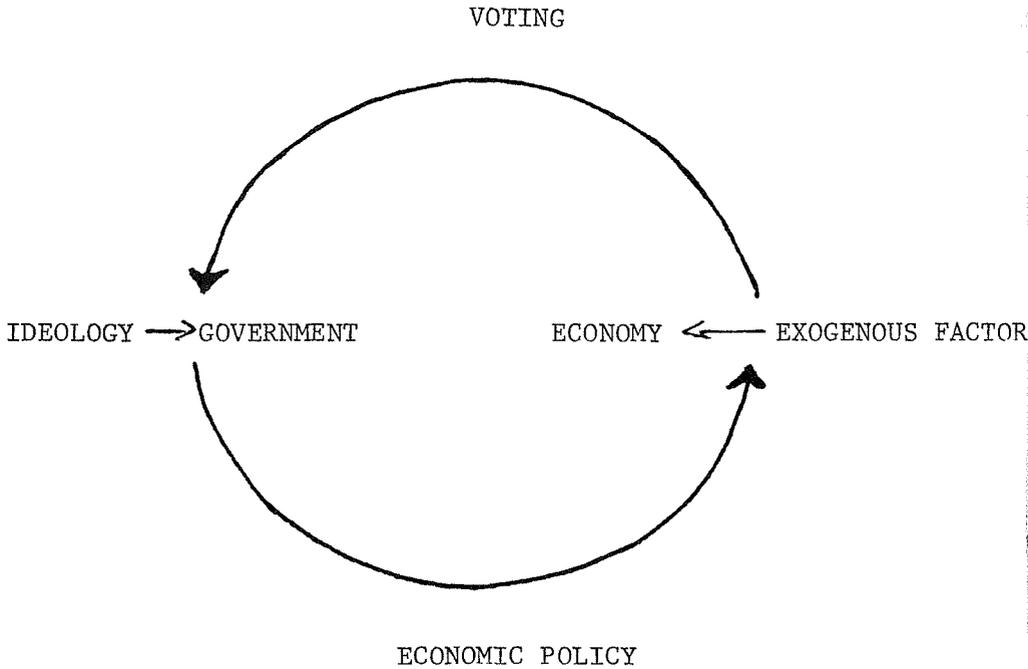
<sup>1</sup>One must mention, however, the pioneering work of Hotelling [13] and Schumpeter [26] in which the interrelationship between government economic policy and the economy is explicitly recognized.

<sup>2</sup>While theories of government behavior are a relatively recent development, significant advances have been made. See for instance Downs [11], Riker [23] and Kramer [17]. For a survey and discussion of related issues, see Frey and Lau [12].

<sup>3</sup>In the rest of the paper, "government" is used interchangeably with "party."

<sup>4</sup>Some such examples are Tinbergen [28] and Theil [27]. Note also that by not assuming a global social welfare function, one also avoids the problem of its nonexistence as raised by Professor K. J. Arrow [2].

Figure 1



possible alternative social states based on its ideology. One may call this the *social welfare function as perceived by the government*.

Knowledge of government preference will enable more reliable econometric predictions to be made. The economic policy instruments are explicitly recognized as endogenous variables of the economic system, depending perhaps on the lagged values of the goal variables. Such a forecasting model dispenses with often arbitrary conjectures on the possible actions of alternative governments under alternative exogenous conditions.

### I. THE HYPOTHESIS AND CONCEPTS

#### *A General Hypothesis of Government Behavior*

The behavioral hypothesis is that the government endeavors to maximize its utility,  $U$ , over all future periods of time.<sup>5</sup> Formally

<sup>5</sup>It should be noted that our approach here is perfectly deterministic but can be generalized to apply to a model with uncertainty.

$$(1) \quad \max U = \int_0^{\infty} F(t)a(t)dt,$$

where  $F(t)$  is the instantaneous utility flow or felicity at time  $t$ . It is assumed that the government, like individuals, values future less than present felicity; hence felicity at any moment is discounted by the factor  $a(t)$ .<sup>6</sup> Felicity is a function of ideological satisfaction,  $I$ , and instantaneous approval,  $A$ .<sup>7</sup> Both  $I$  and  $A$  are so defined that their values always lie between 0 and 1. The felicity function is thus

$$(2) \quad F(t) = F[I(t), A(t), t]$$

The third argument,  $t$ , is included to allow for the possibility of an exogenous shift in the felicity function over time. It is assumed that the felicity function is non-decreasing and concave in  $I$  and  $A$ . If  $\frac{\partial F}{\partial I} = 0$ , then the government is just interested in approval;<sup>8</sup> and if  $\frac{\partial F}{\partial A} = 0$ , the government is oblivious to whether its actions are popular with the voters or not. The felicity function hence includes vote maximization and the maximization of ideological satisfaction as special cases.<sup>9</sup>

*Goals and the Economic System*

The politically relevant issues  $y_1, y_2, \dots, y_n$  or goals influence both the approval of the electorate and the ideological satisfaction of the government. Issues are only relevant to either the party or the voters if first, they are perceived; and second, they arouse a minimum level of emotions; and third, there exists a party differential. However, it is possible for the issues to influence only the ideology function but not the approval function and vice versa. If only economic goals are considered, they may be composed of the five "traditional" goals of an economy, namely: price stability, adequate economic growth, equilibrium in the balance of payments, "just" income distribution, and full employment. Other goals, such as a high level of expenditures for urban development, may also be included. It may be

<sup>6</sup>It is customary to assume that at a constant rate of felicity, the utility integral converges. This implies that the integral over time of the discount factors must be finite, i.e.

$$\int_0^{\infty} a(t)d(t) < \infty .$$

<sup>7</sup>Rothenberg [24] suggests that politicians are not solely concerned about votes, but also enter political life in order to put ideas into practice. This is the same motivation for our ideological satisfaction function.

<sup>8</sup>This is the pure Downsian case. See Downs [11].

<sup>9</sup>Pure maximization of ideological satisfaction may take place in a totalitarian society. Even in that case, however, there may be increasing costs involved in the suppression of discontent such that unconstrained maximization of ideological satisfaction may be unsustainable if the felicity function is basically incompatible with the approval function.

argued that some of these goals are not final but intermediate (e.g., price stability); that some are constraints rather than goals (e.g., the balance of payments equilibrium). Moreover, this list does not even cover other important economic goals such as maximum consumption per head. However, we shall not discuss the normative question of what should constitute the set of final goals here.

All goals are defined such that the ideology or approval function is always strictly monotonic (though not necessarily all of the same sign) in the values of the goals (e.g., more employment is always better than less).<sup>10</sup> The resource limitations within which the economy can operate is given by a production possibility frontier of the following form

$$(3) \quad G(y_1, y_2, \dots, y_n; z(t), t) = 0,$$

which is assumed to be strictly convex in  $y$ . A vector of exogenous, fixed, or inherited factors entering nonrandomly into the relationship and beyond the control of the government is  $z(t)$ . The possible shift of the relation over time which may be interpreted as the influence of technical progress is indicated by  $t$ .

The government does not control the goals directly, but must use instruments (such as monetary, fiscal or wage policy) for that purpose. It is assumed that the government always has sufficient instruments to control the goals within the resource limitations.<sup>11</sup> Hence any particular choice of goals within the production possibility set can be accomplished by a suitable choice of the instruments. As is well known, the dependence of goals upon instruments is empirically estimated in the reduced form of macro-econometric models.

#### *The Ideology Function*

Ideological preferences of the government may be represented by the relative valuation attached to the different political issues or goals.

$$(4) \quad I = I(y_1, y_2, \dots, y_n, t)$$

For the one-party model, it can be assumed, without loss of generality, to be nondecreasing and strictly concave in the goals.<sup>12</sup>

<sup>10</sup>Other alternatives are possible but the present one appears most natural and nonarbitrary. See Frey and Lau [12].

<sup>11</sup>On the relationship between goals and instruments compare Tinbergen [28]. It is also assumed that the desired levels of the goals within the production possibility set can be instantaneously attained by the government for the sake of simplicity.

<sup>12</sup>There exist, of course, preference orderings that cannot be represented by a real-valued function. However, only preference orderings that can be represented by a real-valued function are considered here.

The ideology of a party is solemnly declared and constantly repeated in party programs and manifestos. Content analysis of speeches and writings by party leaders or party ideologists provides further information. The construction of an ideology function may not always be easy as there are circumstances in which it is advantageous for a party to obscure it.<sup>13</sup> Nevertheless, there are definite features in each party's ideology which differentiates it from the other parties.

In the empirical determination of the ideology function care must be taken not to infer ideology simply from what ruling parties actually do. It is one of the contentions of this paper that the government may deliberately deviate in its actions from its most preferred ideological positions. Information on ideology must hence be gathered independently of actual behavior. However, it should be noted that given the present formulation of the problem, it is possible to identify the weights used in the ideology function from actual government behavior alone if the approval function and the economic system constraints are known a priori or have been estimated.

*The Approval Function*

The popularity of the government as reflected, e.g., in regular polls, depends on how well it performs with respect to the various goals. Without concerning ourselves with the problem of aggregating individual approvals into a social approval function, it is assumed that there exists a well-defined aggregate instantaneous approval function which depends only on the instantaneous levels of the various goals and possibly time, that is

$$(5) \quad A = A(y_1, y_2, \dots, y_n, t).$$

It is again assumed to be strictly concave with diminishing returns; however, the marginal approvals need not all be positive.

The approval function can be estimated empirically using regularly taken polls. Even the most casual observation of political life suggests that there is indeed quite a close interrelationship between economic conditions and government popularity. There are, however, few studies concerned directly with this relation, although there have been some studies focused on the influence of the business cycle upon elections outcomes, e.g., L. H. Bean [3, 4], J. Åkerman [1], W. Kaltefleiter [14] and G. H. Kramer [18].<sup>14</sup> One must emphasize that our concept of the approval function is an instantaneous one and depends only on instantaneous achievement and/or performance. On the other hand, election outcomes and voting

<sup>13</sup>See Downs [11]. The party may wish to appear different to different blocs of voters.

<sup>14</sup>These studies will be discussed in more detail in a later section.

behavior are generally determined not by instantaneous approval but by some time average of past approvals. Thus the approval function is more akin to a Gallup Poll than to actual ballot behavior.

The politometric estimation of approval can be supplemented by sample surveys. In many countries such surveys are continuously undertaken and published on a journalistic level. Moreover, governments in fact perceive the existence of an approval function and conduct open or secret surveys to find out the opinion of voters on certain issues.

There are also available more refined scientific public opinion surveys. Such studies are of particular interest because they can explore the voters' preferences through interlinking questions. An example has recently been published by E. Mueller [21] who establishes a rank ordering of public preferences with regard to government programs which give useful indications about the approval function.<sup>15</sup>

Finally, it should be noted that changes in the levels of the goals may be caused by the exogenous factors  $z(t)$  or random shocks as much as by the control variables. Nevertheless, any such improvement or deterioration in the performance of the economy is generally attributed to the government and is reflected in the instantaneous approval of the government by the voters. Thus it is conceivable that a government may become unpopular because of unfavorable external circumstances despite efforts to maximize approval.

#### *Good-will*

The good-will which the government has accumulated at any given time is defined as the summation of all past instantaneous approvals, appropriately depreciated.

$$(6) \quad \pi(t) = \int_{-t_{\infty}}^t A(t') dt' - \int_{-t_{\infty}}^t \int_{-t_{\infty}}^{t'} A(z) m(t'-z) dz dt' + \pi_{-t_{\infty}}$$

It is natural to assume that the further in the past that a government action is taken, the less weight it carries in the voters' decision, other things being equal. An alternative possibility is that the voters remember all past government actions equally well, but only over a finite period. In general, the importance attached to a given government action taken at time  $z$  by a voter at time  $t$  may be represented by

<sup>15</sup>See also Campbell, Converse, Miller and Stokes [9].

$(1 - \int_0^{t-z} m(t') dt')$  where  $m(t')$  is the mortality density function.<sup>16</sup> The fraction of approval generated during the small interval  $dz$  around the time point  $z$  that is forgotten by the voters at time  $t$  is represented by  $m(t-z)dz$ . Alternatively, it can be interpreted as the probability distribution of time that the voters will take into account a given action taken at  $z$  when evaluating the past actions of the government at time  $t$ .

The first term of Equation (6) sums up the approval enjoyed by the government from some distant time in the past  $(-t_\infty)$  to the present. Two other alternatives are possible. The first is to start anew with every election. The second is to start anew every time the government changes hands. Both of these can be considered as special cases of our formulation embodying different assumptions on the mortality density function. The second term sums the quantity of approval of each "vintage" that has disappeared through depreciation. The last term gives the good-will of the government at the initial instant; it is assumed to be zero.

Differentiation of Equation (6) with respect to time yields the change in good-will at any given instant

$$(7) \quad \dot{\pi}(t) = A(t) - \int_{t_\infty}^t A(z)m(t-z)dz$$

Equation (7) will be of particular importance in the subsequent analysis, as it represents the transition equation of a Pontryagin type optimization problem. However, one should note that Equation (7) is an integro-differential equation and is in general not directly amenable to Pontryagin methods without further transformation. In order that Pontryagin methods may be applied, it is necessary to solve Equation (7) for  $\pi(t)$  in terms of  $A(t)$  and integrals involving  $\pi(t)$  and the convolutions of the mortality density function. This can be accomplished by either Fourier Transforms or the method of iterated kernels.

The distinction between good-will and approval cannot be overstressed. Good-will may be compared to capital stock whereas instantaneous approval may be compared to investment. Voter decisions are influenced by good-will rather than approval, although current approvals may carry heavy weights. In this connection, Berelson, Lazarsfeld and McPhee [5] have long pointed out that "votes and opinions on issues are not comparable at all in stability. For example, if votes are

<sup>16</sup>The mortality density function is commonly employed in the context of capital theory but appears appropriate in this application. Among its properties are the following:

- (i)  $m(0) = 0$
- (ii)  $1 \geq m(t) \geq 0$  for all  $t$
- (iii)  $\int_0^\infty m(t) dt = 1$

considered as average or summary expressions of a multitude of smaller judgments moving in different directions, then obviously the average is more stable than the parts.”<sup>17</sup>

*Voting Decisions and Election Outcomes*

In a democracy, the government will be reelected if it receives a sufficiently large share of votes,  $\bar{V}$ . The size of  $\bar{V}$  required depends on institutional factors such as the specific election rules and the number of competing parties. For any election at time  $NT$ ,  $N$  an integer, the actual vote share received by the government  $V(NT)$  must be larger than or equal to  $\bar{V}$ :<sup>18</sup>

$$(8) \quad V(NT) \geq \bar{V} .$$

In this paper, the working assumption of a purely passive electorate is made. The individual voters, just as the individual consumers, are presented with a set of alternatives over which they, as individuals, have no control. They can make a choice amongst the alternatives offered, but generally do not influence individually the range of alternatives that are presented. This rules out any coalition games of the Riker [23] variety.

Additional assumptions concerning the voter decision making process are necessary. As in a previous paper (Frey and Lau [12]), the actual vote share received at time  $NT$  is assumed to equal the “average” good-will enjoyed by the government over the whole past (appropriately depreciated)

<sup>17</sup>Berelson, Lazarsfeld and McPhee [5, pp. 16-17].

<sup>18</sup>It is possible to allow for the case that the government not only discounts future felicities derived from being in office, but also takes future election requirements less and less seriously. This can be expressed by applying another set of discount factor  $\beta(t)$ , which decreases monotonically with time. The following conditions for reelection at subsequent election dates are then obtained:

$$\begin{aligned} V(T) &\geq \bar{V} \beta(T) \\ V(2T) &\geq \bar{V} \beta(2T) \\ \dots &\dots\dots\dots \\ V(NT) &\geq \bar{V} \beta(NT) \end{aligned}$$

In general, this is a planning horizon problem. How far ahead does the government look? Note that this is a separate problem from that of discounting future utilities although it may be difficult to distinguish between both factors simultaneously.

$$(9) \quad V(NT) = \frac{\pi(NT)}{\int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'') dt''] dt'}$$

The expression in the denominator is a weight which takes forgotten good-will into account by attaching relatively greater importance to the more recent past. It also represents the maximum accumulated good-will that the government may have at any time. If the government has, over the past, always had the same percentage of voters supporting it, the vote share at election NT turns out to be equal to this constant approval rate, i.e., if  $A(t') = \bar{A}$  for  $-t_\infty < t' < t$ , then

$$(10) \quad V(NT) = \frac{\pi(NT)}{\int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'') dt''] dt'} = \bar{A} .$$

Combining Equations (5) to (7) gives

$$(11) \quad \frac{\pi(NT)}{\int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'') dt''] dt'} \geq \bar{V} ,$$

$$\pi(NT) \geq \bar{V} \left[ \int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'') dt''] dt' \right] ,$$

defining excess good-will,

$$(12) \quad \hat{\pi}(NT) \equiv \pi(NT) - \bar{V} \left[ \int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'') dt''] dt' \right] \geq 0 .$$

Equation (12) represents the re-election constraint.<sup>19</sup>

The voter decision-making process just described may be referred to as a pure "backward looking" voting strategy. The voters are primarily motivated by their past experience with the government and either ignore future performance

<sup>19</sup>In this paper, it is assumed that the government always wants to be reelected. Moreover, there is perfect information and hence perfect certainty.

completely or have neutral expectations concerning government performance, namely, "tomorrow will be just like today." Note that in this particular case, what the government declares that it would do if elected has absolutely no influence on election outcomes.

In practice, as far as a government already in power is concerned, the assumption of a pure "backward looking" electorate may not seem so unrealistic. Many studies of voting behavior (e.g., Campbell, Converse, Miller and Stokes [9], Key [16] and Kramer [18]) have found that the voters do not generally vote for a party or a program as such, but only vote against the incumbent party if its past performance up to the date of the election is judged unsatisfactory. The "backward-looking" electorate assumption also justifies the application of a one-party model to a world with many parties as long as it is feasible for the incumbent party to remain in power by appropriate maneuvering of the control variables.

Other modes of voter decision making processes are possible. Only two are taken up here:

(1) *Pure "forward-looking" electorate.* The pure "forward-looking" voter is one who "lets bygones be bygones." For these votes, good-will is the integral of discounted future approvals, which depends on the time paths of the control variables proposed by the government in future.

$$(13) \quad \pi(t) = \int_t^{\infty} A(\hat{y}_1(t'), \dots, \hat{y}_n(t')) \cdot \beta(t' - t) dt'$$

where  $y_1(t')$ s are the promises and  $\beta(t')$  is the discount factor. It is assumed that the  $y_1(t')$ s do lie on the production possibility surface, that is, they are attainable promises. However, only a completely naive electorate will take the campaign promises at their face value. Frequently, they discount the promises of the government based on a comparison of the past record of fulfillment of promises. This implies that good-will will be a function of past performance, past promises, future promises all appropriately depreciated, and discounted.

For such a forward-looking electorate, all parties have to be taken into account explicitly in general. However, in the special case of a completely incredulous electorate, the expectations of the future may be based completely on past experience. Thus it is essentially reduced to the "backward-looking" electorate. Kramer [18] has applied such an adaptive expectations model with respect to voting behavior: Note that from a purely empirical point of view the adaptive expectations hypothesis cannot be distinguished from the "memory depreciation" hypothesis described earlier.

(2) *Pure "inertial" electorate.* There are two kinds of voter inertia: one related to party loyalty and the other related to incumbency inertia. If all voters are loyal to their respective parties and have the same vote participation rates and birth rates (assuming that the children also adhere to their parents' party), then the problem is straightforward. Whoever is in the majority will be reelected for every election and the government needs only maximize its felicity at every point without being bound by the election constraints. Secondly, if most voters favor the incumbent, then the same situation as the first prevails. Generally speaking, though, the incumbent does have a decided edge in the election and is returned to office as long as his performance is judged satisfactory. As discussed earlier, this amounts to a one party model with a "backward-looking" electorate for the duration during which the party is in office. As external conditions become unfavorable and another party is elected into office, the edge shifts to the other party and once more a one party "backward-looking" electorate model applies.

No doubt there exist other decision processes. It is believed that in general the decision process of the voter involves all of the above considerations; that is, it is a mixture of the three processes outlined.

## II. THE FORMAL ONE-PARTY MODEL AND ITS SOLUTION

Taking the various parts of the one-party model together, the formal problem may be stated as follows:<sup>20</sup>

$$\max U = \int_0^{\infty} F(I(y_1, y_2, \dots, y_n), A(y_1, y_2, \dots, y_n)) \cdot a(t) dt \text{ subject to the production possibility frontier } G(y_1, y_2, \dots, y_n; z(t), t) \geq 0,$$

the transition equation

$$\begin{aligned} (14) \quad \dot{\hat{\pi}} &= \hat{\pi} - \bar{V} [1 - \int_{-\infty}^t m(t-t') dt'] \\ &= A(t) - \int_{-\infty}^t A(t') m(t-t') dt' - \bar{V} [1 - \int_{-\infty}^t m(t-t') dt'] \\ &= [A(t) - \bar{V}] - \int_{-\infty}^t [A(t') - \bar{V}] m(t-t') dt' \end{aligned}$$

the reelection constraint

<sup>20</sup>Departing from the more general formulation, it is assumed here that the felicity, ideology and approval functions are autonomous, i.e., themselves independent of time.

$$\hat{\pi}(NT) \geq 0 \quad N = 1, 2, \dots$$

and the initial condition

$$\hat{\pi}(0) \geq 0 .$$

The special peculiarity present in this political application is that the constraints are only relevant at discrete points of time, thus violating the Kuhn-Tucker constraint qualification. Hence it is not possible to apply directly either the classical calculus of variations or the Maximum Principle of Pontryagin and associates [22]; their approach requires a constraint set with an *interior* which does not exist in the present problem.<sup>21</sup>

Nevertheless, it is possible to characterize the solution to this optimization problem, although, in common with other optimization problems involving inequality (non-negativity) constraints, it is not possible to give an explicit and analytical characterization of the solution.

#### *Feasibility*

It must first be established that there exists *at least* one feasible program, i.e., it must be possible to choose the goals  $y_1(t), y_2(t), \dots, y_n(t)$  such that no constraints are violated at any time.

Since there exists no intertemporal relationship among the  $y_i$ 's in the model, it follows that any program which maximizes  $A(t)$  at every point of time also maximizes the good-will of the government  $\pi(t)$  at every point and hence also  $\hat{\pi}(t)$ . In particular, it maximizes  $\pi(t)$  at the nodal points (election dates)  $t = NT$ , where  $N = 1, 2, \dots$  (an integer). Thus feasibility is established if the path of  $\pi(A_{\max})$  (with  $A$  maximized at every point) satisfies the constraints at the nodes, i.e., it is never lower than

$$\pi(NT) = \bar{V} \left[ \frac{\pi(0)}{V(0)} + \int_0^{NT} \left[ 1 - \int_0^{r'+t} m(t'') dt'' \right] dt' \right]$$

<sup>21</sup>Several natural economic counterparts of the above optimization problem immediately suggest themselves. They are:

- (i) Optimization problems associated with spatially or temporally fixed delivery quotas;
- (ii) Optimization problems of factory managers and administrators who are required to fulfill certain targets at specified points in time. (This is probably especially relevant for state enterprises in socialist or communist economies.)
- (iii) Inventory problems.

at  $t = NT$ . Alternatively,  $\hat{\pi}(A_{\max}) \geq 0$  at  $t = NT$ ,  $N$  an integer. It is a specific characteristic of this political model that feasibility is *not* violated if  $\pi(A_{\max})$  is below the limit  $\pi$  at points in between election dates.

It must be noted that in the case of nonfeasibility the optimization problem as developed in this section is changed: it is necessary to introduce a second party explicitly. Feasibility depends on exogenous conditions as well as the approval function.

*Optimality*

Two cases may be distinguished:

(1) The election constraints are never binding. The optimal government policy consists in the unconstrained maximization of its felicity, subject to the economic system at every point of time:

$$\begin{aligned} \max U &= \int_0^{\infty} F(y_1, y_2, \dots, y_n, A(y_1, y_2, \dots, y_n)) a(t) dt \\ &= \max F, \text{ for every } t. \end{aligned}$$

The optimal path, if one exists, is (due to the convexity assumptions imposed on  $F$ ,  $A$ ,  $l$ , and  $g$ ) unique. This may be obtained using Pontryagin [22] methods. In fact, if a stationary environment prevails, i.e.,  $z(t)$  is a constant and  $G$  is independent of time, it can be shown that the optimal policy converges onto a steady state path. A real life situation in which the election constraints are never binding may have existed for the Democratic party in some Southern states.

(2) Some election constraints are binding. In general it must be assumed that the government is sometimes forced to depart from its felicity maximizing policy in order to stay in office, while at other times it is not. For an infinite horizon problem with arbitrary  $z(t)$ , an optimal program may not even exist. However, if an optimal policy exists for the unconstrained counterpart of the optimization problem, then there also exists an optimal program for the constrained problem if there exists at least one feasible program. It is possible to state the following characterization of the structure of the optimal program of the government for a finite horizon problem. It is assumed that the government looks ahead for  $N$  elections and that the unconstrained optimum is not feasible.

*The Critical Point Property.*<sup>22</sup> A critical point of a program  $y(t)$  is said to occur at the election at time  $NT$  if  $\pi(NT) = 0$ ,  $N$  an integer  $\leq N$ . A feasible program is said to have the *critical point property* if  $\hat{\pi}(NT) = 0$  for some  $N$ .

<sup>22</sup>See Manne [19, 20, especially Ch. 11], for an excellent discussion of some related dynamic optimization problems.

*Critical Point Theorem.* There is an optimal program (not necessarily unique) which has the critical point property if the unconstrained maximum is not feasible.

This theorem is intuitively obvious: if the unconstrained maximum is not feasible, then some constraints must be effective; that is,

$$\hat{\pi}(NT) = 0 \text{ for some } N.$$

It is extremely important as it enables one to confine search of the optimal program among those with the critical point property.

Given a set of critical points corresponding to a feasible program, there exists a program which yields the highest value of the utility integral. A piecewise application of either Pontryagin's maximum principle or the Euler-Lagrange equation for optimization between adjacent critical points will yield the undominated program given the set of critical points, as the initial and final conditions are both given. (Intertemporal additivity of felicities is a crucial assumption!) There will in general be a large number of undominated programs with the critical point property, with an upper bound of  $2^{(N-1)}-1$ . However, through the simple property of intertemporal additivity of felicity one can further restrict the number of programs that should be considered.

*Admissibility.* An undominated critical point property program is said to be admissible if there exists no other program whose set of critical points are completely contained in the set of the former.

*Admissibility Theorem.* There exists an optimal program which is admissible. This theorem follows from the fact that an unconstrained maximum is always greater than or equal to a constrained maximum between the same end points and with the same initial and final conditions.

Thus, for the purpose of finding an optimal program, one needs only consider the class of admissible programs. One can compute the values of the utility integral corresponding to each admissible program and then choose the one with the highest value. This must then be the optimal program. Note that the critical points in the optimal program have the interpretation of "critical elections" in the sense of Key [16].

*An Algorithm.* An efficient backward recursive algorithm which computes the value of at most  $2^{(N-1)}-1$  programs may be used.<sup>23</sup> A sub-optimization operation  $S(N_1, N_2)$  is defined as the rule which gives the optimal program between  $N_1$  and

<sup>23</sup>A similar forward recursive method may be developed.

$N_2$  subject to the conditions that  $\hat{\pi}(N_1 T) = 0$ ,  $\hat{\pi}(N_2 T) = 0$ , except for  $\hat{\pi}(0) = \hat{\pi}_0$  and  $\hat{\pi}(NT)$  which is not given. The algorithm can be conveniently described in terms of S.

(1) Set  $K = 1$ ;  $C(I, J) = 0$ ;  $I, J = 1, \dots, \bar{N}$

(2) Set  $N_1 = \bar{N} - K$ ;  $L = 1$

(3) Calculate  $S(N_1, \bar{N})$ . If feasible do (4). Otherwise set  $K^*(K) = 0$ . Do (8).

(4) Check if  $N_1 = C(I, J)$  for any  $I < K$ ,  $K^*(I) = 1$  and  $J < \bar{N} - 1$ . If true do (5). Otherwise calculate  $S(0, N_1)$ . If feasible do (5). Otherwise set  $N_2 = N_1$ ;  $N_1 = 0$ ; do (6).

(5) Set  $K^*(K) = 1$ . A partially admissible program has been obtained (subject to elimination when the whole set of partially admissible programs have been obtained). Do (8).

(6) Set  $N_1 = N_1 + 1$ . Check  $N_1 < N_2$ . If true do (7). Otherwise set  $K^*(K) = 0$  and do (8).

(7) Calculate  $S(N_1, N_2)$ . If feasible, set  $C(K, L) = N_2$ ;  $L = L + 1$ . Do (4). Otherwise do (6).

(8)  $K = K + 1$ ; if  $K < \bar{N}$  do (2). Otherwise end.

One proceeds in this way until all admissible programs are obtained, i.e., until  $K = \bar{N} - 1$ .

For the infinite horizon problem, it has not been possible to verify the existence of an optimal program, let alone finding the program itself. However, under the conditions of stationary, (or periodic) environment, a solution appears feasible.<sup>24</sup>

### Special Cases

Without further specification of the felicity, ideology, approval, and mortality density functions and the economic environment, no more can be said about the optimal policy, except that it can be efficiently constructed using the algorithm outlined in the section on optimality. The foregoing also provides a constructive proof of the existence of the optimal policy.

<sup>24</sup>The basic argument used there is that the optimal program will consist of repetitions of an optimal sub-program for a finite horizon.

For the special case that all the election constraints are binding, either because of adverse external environment or because of incompatibility of the ideology and approval functions, it is easy to see, without the complication of mathematics, that good-will, as well as the control variables which influence approval, may undergo fluctuations within each election interval. Specifically, good-will will be lower in the middle of an election interval rather than on both ends. The magnitude and time phasing of these fluctuations depend very much on the mortality density function and the external environment.<sup>25</sup>

Other lags which have been neglected may also be relevant, e.g., the voter recognition and reaction lag, the government recognition and reaction lag, and the response lag of the goals to policy instruments. Abstracting from all these factors, one may infer that in a country without one single dominant ideology subscribed to by a large majority of the population, it is possible that the political system (as well as exogenous disturbances) may generate and reinforce economic cycles. A change of policy that is more popular rather than ideologically sound from the party's point of view is more likely as the election period draws near. Unpopular actions will most probably be postponed. It is also evident that the less the population identifies with the ideology of the ruling party, the larger, *ceteris paribus*, are the politically generated fluctuations. Finally, the good-will accumulated by the party in power tends to increase towards election time and decrease after the election.

### III. SOME EMPIRICAL EVIDENCE

In principle, one can estimate an econometric model of government behavior and test it against alternative theories of government behavior. This appears to be a lengthy task and has not been done. Instead, some casual observations are offered as evidence that the basic behavioral hypothesis proposed is consistent with observed government behavior.

#### *Voting Behavior*

Many students of elections have noted that the voting behavior of the electorate is influenced by economic factors preceding the election. A notable contribution is by Bean [3, 4] who sees the business cycle as by far the most important predictor for the outcomes in both presidential and congressional elections in the United States. He even derives an elasticity describing the average percentage swing against the ruling party when national income changes unfavorably. Akerman [1] also finds the same kind of dependence for England, Germany, and, less clearly so, for Sweden. More recently, an important contribution is made by Kramer [18] who explains the short-term variations in the share of votes received by the two major parties in United States elections. He succeeds in relating

<sup>25</sup>Several examples with different mortality density functions have been worked out and graphed in Frey and Lau [12].

about half of the observed variations to the influence of economic factors. The explanatory variables are the growth rates of real and nominal income, the growth rate of prices (inflation) and the rate of unemployment. He finds that a higher income growth rate benefits the ruling party, an increase of inflation is favorable to the opposition while the influence of unemployment is not significant. Kaltefleiter [14] has analyzed the elections in Germany during the Weimar Republic and for the post-World War II period, and correlates the votes for Nazis and the Communists as a function of the unemployment rate using both time-series and cross-section data. These studies clearly demonstrate a close relationship between government performance in fulfilling the goals and good-will, and that the approval function used in the present model has a counterpart in real life.

#### *Good-will of the Government*

Good-will at any instant is best represented by the voting intentions, as distinct from instantaneous approval, of the electorate. It is a well known fact that the party in power almost always loses good-will with the voters between the elections, as demonstrated by generally lower approval ratings and voting intentions. But almost invariably the incumbent party regains the good-will as election time draws near, although this gain may or may not be sufficient to secure re-election. Berelson, Lazarsfeld and McPhee [5] find that between 1944 and 1948, in the "representative" community of Elmira, the Republicans started with a 61 percent good-will which rose to a maximum of 72 percent by June, 1948 and then gradually declined to 61 percent by election time. In the presidential election of 1968 there is confirmation of a similar phenomenon for the nation as a whole. It has also been pointed out that the tendency of the incumbent party to lose a few seats to the opposition especially during mid-term elections appears quite uniform.<sup>26</sup>

Analyses of the voting intentions of British voters have revealed a similar pattern in the general elections of 1955, 1959 and 1964.<sup>27</sup> In addition, the good-will ratings are also correlated to the indicators of economic activity such as gold plus dollar reserves, unemployment, and weekly income.

#### *Government Behavior*

In the political science literature it is not clearly perceived that the government uses its instruments to enable it to win an election. Instead, the role of a "campaign strategy" is emphasized.<sup>28</sup> In the context of our model, a campaign

<sup>26</sup>Key [15, p. 554].

<sup>27</sup>Butler [6, p. 11], Butler and Rose [7, p. 40], Butler and King [8, p. 15].

<sup>28</sup>See Butler and King [8, Ch. 3-6], for a discussion.

involves only the efficient dissemination of information both pertaining to the past and the projected future.<sup>29</sup>

Butler and King [8] write: "Although politicians' ideas about voting behavior may have been rudimentary, most of their major decisions were carefully and rationally considered."<sup>30</sup> This confirms to a certain extent our rationality postulate for the government. In addition, many scholars have noted that politicians are very much aware of the timing of their actions.<sup>31</sup> Actual examples are legion. For instance, it is well known that the incumbent government is generally reluctant to raise taxes in an election year.

#### *Nonfeasibility*

There are circumstances under which no feasible program exists. Under this situation any government in power will inevitably fail. Such an unstable situation caused by unfavorable external conditions might have occurred in Austria and Germany after the economic crisis of 1929.

#### *Conclusion*

Finally, one must add that not all political cycles are caused by economic factors, and vice versa. The important point is the explicit recognition of their mutual interdependence.

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<sup>29</sup>It is also evident that during the course of a campaign, the projected programs offered by a party can undergo changes. This possibility is neglected.

<sup>30</sup>Butler and King [8, p. 56].

<sup>31</sup>E. g., Butler and King [8, pp. 75, 81-83]; Key [15, p. 467].

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